Pharmaceutical Kinetics: Analyzing Drug Concentration and Effectiveness

Name: _____ Date: _____

Context

A pharmaceutical company is developing a new time-release medication for pain management. After a patient takes the medication orally, the concentration of the drug in the bloodstream (measured in milligrams per liter) can be modeled by the function:

$$C(t) = \frac{20t}{t^2 + 4}$$

where t is the time in hours after administration, with $0 \leq t \leq 12$.

The effectiveness of the drug depends not only on its concentration but also on how quickly the concentration is changing. Doctors need to understand the rate at which the drug enters and leaves the bloodstream to determine optimal dosing schedules and to monitor patient safety.

Part (a)

Find $C'(t)$, the derivative of the concentration function with respect to time. Simplify your answer completely and show all steps.

Part (b)

Calculate and interpret $C'(1)$ and $C'(4)$. What do these values tell us about the drug concentration in the bloodstream at these specific times? Include proper units in your answer.

Part (c)

Determine when the drug concentration reaches its maximum value. At what time does this occur, and what is the maximum concentration? Show all work and explain your reasoning using calculus.

Part (d)

The therapeutic effectiveness $E(t)$ of the medication depends on both the concentration and the body's sensitivity to the drug, which decreases over time. The effectiveness is modeled by:

$$E(t) = C(t) \cdot S(t)$$

where $S(t) = e^{-0.15t}$ represents the body's sensitivity factor. Find $E'(t)$ and determine the rate at which effectiveness is changing at $t = 2$ hours (when concentration is at its maximum). Express your answer to three decimal places.

Part (e)

A patient's pain threshold requires the drug concentration to be at least 3 mg/L to be effective. Using calculus techniques, determine the time interval during which the drug concentration is above this therapeutic threshold. Then, calculate the **average rate of change** of concentration during this therapeutic window and compare it to the **instantaneous rate of change** at the midpoint of this interval. What does this comparison reveal about how the drug behaves during the therapeutic period?