Solar Microgrid Optimization for Campus Energy Independence

Name: ______________________________ Date: ______________

Context

Riverside High School is designing a solar microgrid to reduce energy costs and carbon emissions. The engineering team has collected data on two types of solar panels: monocrystalline (Panel A) and polycrystalline (Panel B). The available roof space is 2400 square meters on the main building.

Panel Specifications:

Specification	Panel A (Mono)	Panel B (Poly)
Dimensions	$2.0 \text{ m} \times 1.0 \text{ m}$	$1.8 \text{ m} \times 1.2 \text{ m}$
Peak Power Output	400 W	320 W
Cost per Panel	$450	$280
Temperature Coefficient	$-0.35\%/^\circ$C	$-0.42\%/^\circ$C
Expected Lifespan	30 years	25 years

Efficiency Data: Engineers tested Panel A under varying sun angles and recorded efficiency as a percentage of peak output:

Sun Angle from Horizontal (degrees)	Efficiency (%)
15	52
30	73
45	88
60	97
75	94
90	83

The school's average daily energy consumption is 8500 kWh, with peak demand occurring between 10 AM and 2 PM. The available budget is $425,000, and the district requires at least 40\% energy independence within the first year.

Part (a)

During summer, the average daily sun exposure provides 6.5 peak sun hours (when panels operate at or near maximum efficiency). If the school installs 800 of Panel A oriented optimally, calculate:

(i) The total peak power capacity in kilowatts (kW)

(ii) The estimated daily energy production in kilowatt-hours (kWh)

(iii) What percentage of the school's daily energy needs would this configuration meet?

Part (b)

Using the efficiency data provided in the table, create a mathematical model for Panel A's efficiency.

(i) Plot the data points and determine whether a linear or quadratic model is more appropriate. Explain your reasoning.

(ii) Find a quadratic regression model in the form $E(\theta) = a\theta^2 + b\theta + c$, where E is efficiency (as a percentage) and θ is the sun angle in degrees. Round coefficients to three decimal places.

(iii) Use your model to predict the efficiency at a sun angle of 50° and explain what this means in practical terms.

Part (c)

The panels will be installed on a roof with a tilt angle of 35° from horizontal. In December, at solar noon, the sun's altitude angle (measured from horizontal) in this location is 31°.

(i) Using trigonometry, calculate the effective angle of incidence - the angle at which sunlight strikes the panel surface. (Hint: Consider the relationship between the sun's altitude, panel tilt, and the angle between them)

(ii) During summer temperatures, panels operate at an average of 65°C. The reference temperature for peak power ratings is 25°C. Calculate the actual power output of Panel A at this elevated temperature, considering the temperature coefficient of -0.35% per degree Celsius.

(iii) If 600 of Panel A and 400 of Panel B are installed, verify whether this configuration fits within the available 2400 square meters of roof space.

Part (d)

The school must decide on an optimal mix of both panel types to maximize energy production while staying within budget and meeting the 40\% energy independence requirement.

Let x = number of Panel A units and y = number of Panel B units.

(i) Write a system of inequalities representing all constraints: budget ($\$425{,}000$), roof space (2400 m^2), and minimum energy production (40\% of 8500 kWh = 3400 kWh daily, assuming 6.5 peak sun hours).

(ii) Determine which combinations satisfy all constraints by testing the following candidate solutions: $(700, 300)$, $(600, 400)$, $(500, 500)$, and $(800, 200)$. Show your work for each.

(iii) Among the feasible solutions, which configuration provides the maximum total peak power capacity? Calculate this maximum capacity in kW.

Part (e)

Given that none of the tested configurations in part (d) meet all requirements, the engineering team must make a strategic recommendation.

(i) Find a feasible solution that satisfies all constraints using mathematical reasoning. Express your answer as (x, y) and verify all three constraints are met.

(ii) The district is willing to increase the budget by up to $\$50{,}000$ OR reduce the energy independence requirement to 35\% (but not both). Using optimization analysis, determine which option is more cost-effective for maximizing long-term energy production. Consider:

- The lifespan difference between panel types
- The total energy produced over each panel's lifetime
- The cost per kilowatt-hour over the panel's lifespan

Provide a detailed justification for your recommendation, including calculations that compare at least two different configurations under each scenario. Your answer should address both technical efficiency and financial sustainability.
